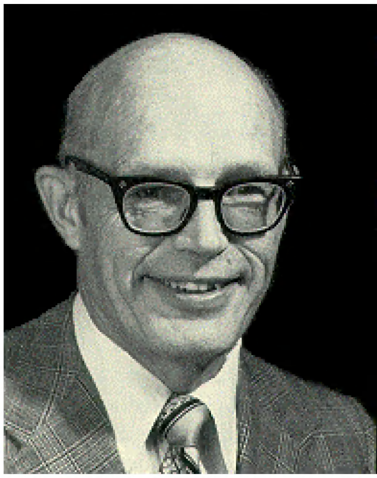
Bently's Corner



by Donald E. Bently
President and Chairman

What happened to the Damping?

It is well known that you can measure the Synchronous Amplification Factor of a rotating machine at some reasonable value, like 4 to 8, and then have the machine get into a bad forward circular instability whip at operating speed and conditions. For such an instability to occur, the Operating Amplification Factor must have gone to infinity.

The obvious question is: "What happened to the Damping?" It will be shown here that the damping is in good shape (more so than you thought) and that it was the Swirling portion of the Quadrature Stiffness that ate your lunch.

Consider a rotating machine that operates above at least one of its self-balance resonance speeds ("criticals"). In order to evaluate its net effective Spring, Damping, Mass, and its Synchronous Amplification Factor, the usual process is to first get the rotor well balanced. Then add an imbalance of a particular weight, radius, and angle at a lateral position that will excite the balance resonance mode of interest.

Next, run the machine from zero speed to a speed well above the balance resonance speed. Document the rpm, amplitude, and phase of the rotative (synchronous) speed on start-up or coastdown across the speed range. Subtract the Slow Roll bow, and plot the data in Polar and Bodé plots.

For this test, the machine should be as nearly as possible in its operating configuration, load, and environment. This applies to its foundation and alignment as well as to the characteristics of its energy source (in the case of a driver element like a gas turbine), or the characteristics of its load (in the case of a pump, compressor, and generator).

Now, let's work an actual problem for a typical rotor system. Here are the pertinent characteristics of a typical machine (except one vital parameter, which will be shown later). The net spring coefficient is 1 million lbs./in. The weight of the rotor is 3300 lbs., and its effective weight (if it were all concentrated in one spot at midspan) is 2316 lbs. Dividing by gravitational acceleration of 386 in./sec.^{2}, the rotor effective mass M_{EFF} is 6 lbs. sec.^{2}/in.

The operating speed is 7640 rpm (which, to make calculations easier, comes out to 800 rad./sec.). From this information, the first balance resonance speed can be found as follows:

$$\omega_{RES} = \sqrt{\frac{K}{M_{EFF}}} = \sqrt{\frac{10^6}{6}} = 408 \text{ rad./sec.}$$

or

$$RPM_{RES} = \left(\frac{60}{2\pi}\right)\omega_{RES} = 3900 \text{ rpm}$$

Figure 1 shows the rotor configuration, its first mode bow shape, and the location of the shaft observing probe.

By looking at the Polar plot or the Slow Roll Compensated Bodé plot (Figure 2), you can work out the Synchronous Amplification Factor, and Observed Damping

D_{OBS}, which is $D_{OBS} = \dfrac{\omega_{RES} \times M_{EFF}}{Q_{SYNCH}}$

If you like to use synchronous damping coefficient zeta, it is:

$$\xi_{SYNCH} = \frac{1}{2Q_{SYNCH}}$$

and, for logarithmic decrement buffs, it is the

$$\text{Synchronous Log Dec} = \frac{\pi}{Q_{SYNCH}}$$

The reason for the subscript adjectives, according to my colleague Joe, is that a rotating machine can do more tricks than a monkey on a 100 foot (30.48 meter) rope. Therefore, don't throw away the subscripts.

From Figure 2, you can figure out the Synchronous Amplification Factor by the half power method, the ratio method, or the phase lag rate method. All these methods work okay, but strictly speaking, you should divide amplitude by speed before doing the half power method. ▶

As you can see, the Q_{SYNCH} is about 7.0. From this you can get

Synchronous Damping Ratio

$$\xi = \frac{1}{2Q} = 0.0714$$

Synchronous Log Dec = 0.449, and

Observed Damping

$$D_{OBS} = \frac{(408)(6)}{(7)} = 350 \text{ lb sec/in}$$

All finished? Not by a long shot. You next need a new graph. Nobody is using this graph yet—except my colleagues—Joe and Agnes — and me. We think it is extremely important because it gives the engineer a clear picture of rotating machinery behavior. Come to think of it, the rotor stability researchers at Cal Tech and Tokyo University also use this graph.

Anyway, this is pretty simple mechanical engineering. To make this graph, you simply write out the total of all the dynamic stiffness terms that you have so far:

Observed Dynamic Stiffness,

$$\overline{K}_{OBS} = K - \omega^2 M_{EFF} + j\omega D_{OBS}$$

This is a vector equation: (1) because I put a bar on top of K_{OBS}, and (2) because, whereas, K and $-\omega^2 M$ take off at 0° and 180°, respectively, the $+j\omega D_{OBS}$ is at right angles (which are much better than left angles, or wrong angles, according to Joe. I am left handed, so I sort of resent that).

Instead of fooling with this vector, break it up into two parts, the Direct happenings at 0° and 180°, and the Quadrature happenings at ±90°. Thus, there are two graphs, K_{DIRECT} and $K_{QUADRATURE}$. (Joe has a big problem with the word "quadrature;" he thinks it has something to do with the number 4. He's in good shape, however, compared to some people who call any term with a $\sqrt{-1}$ imaginary!)

Thus, $\overline{K}_{OBS} = K_{OBS\ DIRECT} + jK_{OBS\ QUADRATURE}$

and $K_{OBS\ DIRECT} = K - \omega^2 M_{EFF}$

and $K_{OBS\ QUADRATURE} = \omega D_{OBS}$

The values are:

$K = 10^6$ lb./in.

$D_{OBS} = 350$ lb. sec./in.

$M_{EFF} = 6$ lb. sec.^{2}/in.

ω is 0 to 1000 rad./sec. (0 to 9550 rpm)

The graphs of the Direct and Quadrature Dynamic Stiffness are shown in Figure 3.

There are more terms in this equation for a typical rotor system, some of them linear, some nonlinear, some symmetric, some nonsymmetric, some having to do with mechanical things, and some to do with aerodynamics or hydrodynamics.

The list of these is impressive, including:

- Support asymmetry
- Rotor asymmetry
- Fluidic inertia
- Gyroscopics
- Swirling ratio of liquids and gasses
- Full and partial rubs
- Coupling lockup
- Lateral effects by torsional action
- Shifts of bearing loading due to alignment
- Shifts of bearing loading due to load.

However, even though these and other terms might be influencing the Dynamic Stiffness of a rotor system, the swirling ratio of the liquids and gasses is clearly the big one to watch.

The swirling ratio, λ (lambda), occurs on nearly all machines. It is the net ratio of the rotation rate of steam, liquid, and/or gasses in the machine around any and all "cylinders inside cylinders." Therefore, bearings (except tilting pad and rolling element bearings), virtually all seals, steam shrouds, all parts of pumps, and oil retainer rings contribute to swirling.

Damping Stiffness is a Dynamic Quadrature term because its term is viscous damping times velocity, and velocity is 90° ahead of displacement motion. The Swirling Dynamic Stiffness is also a Quadrature term, but for an entirely different reason. The Swirling Dynamic Stiffness is the product of three terms: (1) the net swirling ratio, (2) rotative speed, and (3) viscous damping. Swirling liquids and gasses generate an active force on the shaft when they are trapped by a wedge. The wedge is set up behind the constriction formed by the sidewise shaft motion. Because of this, the Swirling Dynamic Stiffness has the form

$$K_{SWIRL} = (-j) \times (\lambda)(\omega_{ROTN}) \times (D) \text{ lb./in.}$$

This term is generally referred to as a "cross spring," or "K_{xy}," or "K_{yx}," or aerodynamic cross coupling. A better term, when dealing with sleeve bearings, is "oil wedge support stiffness."

The dynamic stiffness of viscous damping is $(+j)(\omega_{PREC})$ (D), which is the shaft passively pushing on the fluids or gasses. The dynamic stiffness of the swirling is $(-j)(\omega_{ROTN}) \times$ (D), which is the fluids or gasses actively pushing on the shaft.

These two terms are the principal contributors to the total Quadrature Dynamic Stiffness, so that

$$K_{QUAD} = (-\lambda\omega_{ROTN} + \omega_{PREC})(D) \text{ lb./in.}$$

Therefore, one more parameter is needed to complete the description of this rotor system, the average swirling ratio, λ. A very good assumption for this machine, and for that matter, any turbine, compressor, pump, or generator is about 0.40 to 0.45. (Joe thinks that this is related to regular old oil whirl, and he is right!). Occasionally, rotors whirl at that frequency.

With this new piece of information, Figure 4 shows the actual behavior of this machine, assuming that $\lambda = 0.41$. Note that the horizontal axis is labeled "Precession Speed ω_{PREC}." The Direct Dynamic Stiffness is unchanged because gyroscopics and fluidic inertia, which contribute to the Direct Stiffness, are not immediately important, but there is a totally new Quadrature Dynamic Stiffness term.

The old rule for figuring damping,

$$D = \frac{(\omega_{RES})(M_{EFF})}{(Q_{SYNCH})},$$

does not give the correct result, unless the swirling is zero!!

Here is what works with excellent accuracy on rotating machinery:

$$D = \frac{(\omega_{RES})(M_{EFF})}{(Q_{SYNCH})(1-\lambda)}$$

$$= \frac{(408)(6)}{(7)(1-0.41)} = 593 \text{ lb. sec./in. for this machine example.}$$

The viscous damping is absolutely NOT the 350 lb. sec./in. previously calculated! The good news is that the damping is much higher than that measured by the data from start-up or rundown.

The bad news is that when the rotor is at 7640 rpm (800 rad./sec.), the Swirling Stiffness is:

K_{SWIRL} = -(0.41)(800)(593) = -195,000 lb./in.

As a result, the Quadrature Dynamic Stiffness at balance resonance (first critical) when this machine is at its operating speed of 7640 rpm (800 rad./sec.) is:

K_{SWIRL} @ 3900 rpm
$= -195{,}000 + (593)(408)$
$= -195{,}000 + 242{,}000$
$= +47{,}000$ lb./in.

instead of the 143,000 lb./in. that was observed on start-up.

The Operating Amplification Factor is:

$$Q_{OP} = \frac{(\omega^2_{RES})(M_{EEF})}{K_{QUAD}}$$

$$= \frac{(408)^2 \times (6)}{47{,}000} = 21$$

When the machine is at its operating speed above resonance, this is the vital data necessary; not the Q_{SYNCH} of 7 that appeared on the start-up data.

This behavior is very typical of all rotating machinery. Figure 5 shows the amplitude and phase response as the machine sees its dynamic impedance when it is at operating speed. Compare this to the amplitude and phase observed synchronously in Figure 2 (dashed line in Figure 5) and the necessity of the Direct and Quadrature Dynamic Stiffness plots becomes obvious.

The necessity of specifying the Operating Amplification Factor is also obvious. The example used here has a very slim margin of stability. (When the Direct and Quadrature Dynamic Stiffness terms are both at 0 at the same precession speed, the machine will be whipping at that speed.) With these graphs, it is simple to calculate how much loss of spring stiffness K will introduce instability (Joe says that if this machine goes from 1×10^6 lb./in. to $K = (0.41)^2(800)^2(6) = 645{,}000$ lbs./in., the machine will be in whip.)

Once this knowledge and method comes into general use, the next step will be to take up the higher engineering level problem of controlling the machine parameters to predictably eliminate instability. Some immediate clear cures of instability include: (1) design out the swirling, and (2) if you can't do that, hold the rotor at high eccentricity ratio, like 0.7 or so, in seals and cylindrical bearings with steady-state loading.

For copies of stability papers Dr. Agnes Muszynska and I have published, please contact our Customer Information Center, P.O. Box 157, Minden, NV 89423, (702) 782-3611. ■

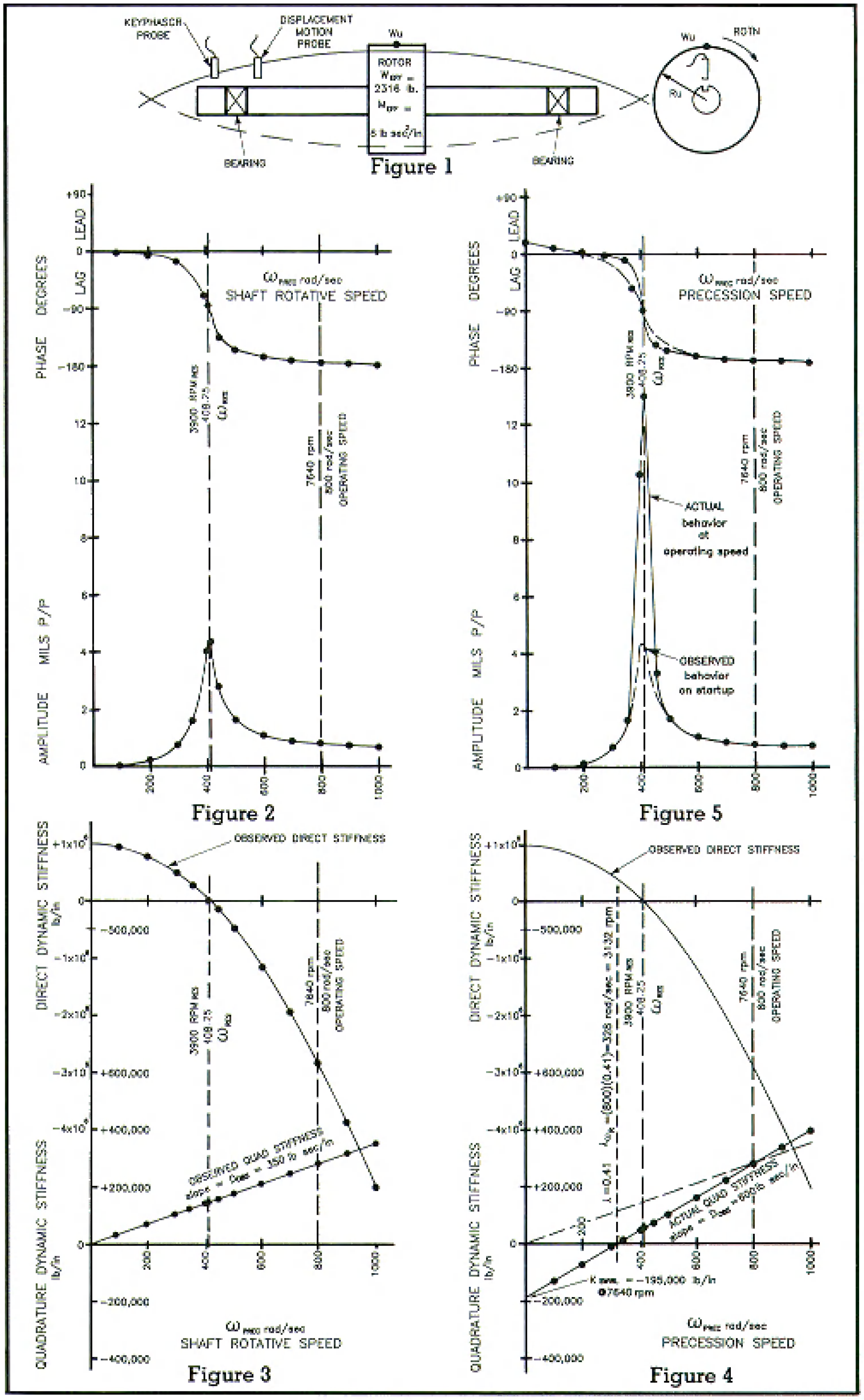


Figure 1

Figure 2

Figure 5

Figure 3

Figure 4